Big Ideas at the Horizon

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Taking horizon content knowledge for teaching (HCK) as knowledge that informs/changes future practice, HCK clearly draws on advances in knowledge in related disciplines. However, research and reflections on teaching practice generate HCK for teaching that is unique to teaching. Knowledge of the big ideas in Number are an example of this type of HCK – it serves to inform teaching to learner's mathematical futures (Ball & Bass, 2009) but it is at the periphery of teachers current content and pedagogical content knowledge. The implications of this for teacher education and practice will be explored in the paper.

Horizon Knowledge for Teaching Mathematics

> With my ears to the ground, listening to my students, my eyes are focussed on the mathematical horizon (Ball, 1993, p. 376).

The notion of horizon knowledge for teaching mathematics was anticipated by Ball in a discussion about the type of knowledge teachers needed to "create and explore practice that tries to be intellectually honest both to mathematics and the child" (1993, p. 377). More recently Ball and Bass (2009) refer to horizon knowledge as

> a kind of content knowledge that is neither common nor specialized. It is not directly deployed in instruction, yet it supports a kind of awareness, sensibility, disposition that informs, orients, and culturally frames instructional practice. We consider this a kind of peripheral vision, or awareness of the mathematical horizon (p. 5)

The horizon metaphor is both helpful and problematic. Teachers need to be aware of the mathematical horizon to be open to the possibilities of extending student thinking into areas of mathematics that they are yet to encounter. However, horizons are relative – if you are lying down between two sand dunes, the horizon is immediate and technically attainable until, of course, you reach the top and a new horizon emerges. Horizons never disappear in this sense but our ability to perceive them may well be clouded by other factors such as opportunity to learn, the daily demands of school life, the crowded curriculum, and the focus on narrow forms of summative assessment.

The mathematical horizon of professional mathematicians is expanding at an exponential rate. By contrast, the mathematics that students are likely to encounter in future years is well documented and unlikely to disappear any time soon. The problem is the extent to which this horizon knowledge is understood by teachers of mathematics, particularly primary teachers and those teaching 'out-of-field', in ways that allow them to see, hear and build on the possibilities afforded by student's thinking.

This situation is compounded by the fact that horizon knowledge is not embedded/represented in curriculum documents and it is only rarely, if ever, addressed in pre-service teaching programs and teacher professional development. This is in marked contrast to the attention given to content knowledge and pedagogical content knowledge, both of which are critically important, but, neither of which necessarily equips teachers with the knowledge and insight needed to recognise and respond in the moment to the possibilities afforded by student thinking to explore mathematical ideas and connections in an intellectually honest and respectful way.

Ball and Bass (2009) contend that this horizon knowledge has four constituent components.

1) A sense of the mathematical environment surrounding the current "location" in instruction.
2) Major disciplinary ideas and structures.
3) Key mathematical practices.
4) Core mathematical values and sensibilities (Ball & Bass, 2009, p. 7).

Big Ideas in Number

The notion of 'big ideas' in mathematics is not new. For example, Charles (2005) defines a 'big idea' as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p. 10). However, there is little agreement about what these are or how they are best represented to support the teaching and learning of mathematics. What might be a 'big idea' from a purely mathematical perspective (e.g., set theory) may not be a 'big idea' from a pedagogical perspective. To serve as underlying structures on which further mathematical understanding and confidence can be built, big ideas need to be both mathematically important and pedagogically appropriate.

From this perspective, big ideas provide "a kind of 'peripheral vision', a view of the larger mathematical landscape that teaching requires" (Ball & Bass, 2009, p.1). Ideally, this includes the capacity to "see backwards, to how earlier encounters inform more complex ones, as well as how current ones will shape and interact with later ones" (p. 10).

The development and use of performance-based, diagnostic tasks in remote Indigenous schools (e.g. Siemon, Enilane & McCarthy, 2004) and the results of two large-scale research projects (e.g. Siemon, 2001; Siemon, Breed, Dole, Izard & Virgona, 2006) prompted the development of the big ideas in number framework presented in Table 1 below. For this purpose, a 'big idea' was defined as an idea, strategy, or way of thinking about key aspects of mathematics:

- without which, students' progress in mathematics will be seriously impacted;
- encompasses and connects many other ideas and strategies;
- provides an organising structure or a frame of reference that supports further learning and generalizations; and
- may not be clearly defined but can be observed in activity (Siemon, 2006).

Table 1
Big Ideas in Number (Siemon, 2006)

By the end of:	Big Idea	Indicated by:
Foundation Year	Trusting the Count	Access to flexible mental objects for the numbers to ten based on part-part-whole knowledge derived from subitising and counting (e.g., know that 7 is 1 more than 6, 1 less than 8, 5 and 2, 2 and 5, 3 and 4 without having to make or count a collection of 7)
Year 2	Place-value	Recognises '10 of these is 1 of those', views tens and hundreds as abstract composite units and larger

		numbers as counts of these units rather than collections of ones (e.g., able to count forwards and backwards in place-value units
Year 4	Multiplicative Thinking	Capacity to work flexibly with both the number in each group and the number of groups (e.g., can view 6 eights as 5 eights and 1 more eight). Recognises and works with multiple representations of multiplication and division (e.g., arrays, regions and 'times as many' or 'for each' idea).
Year 6	(Multiplicative) Partitioning	Ability to partition quantities and representations equally using multiplicative reasoning (e.g., a fifth is smaller than a quarter, estimate 1 fifth on this basis then halve and halve remaining part again to represent fifths), recognise that partitioning distributes over previous acts of partitioning and that numbers can be divided to create new numbers
Year 8	Proportional Reasoning	Ability to recognise and work with an extended range of concepts for multiplication and division including rate, ratio, percent, and the 'for each' idea, and work with relationships between relationships
Year 10	Generalising	Capacity to recognise and represent patterns and relationships in multiple ways including symbolic expressions, devise and apply general rules

Each of the big ideas above is supported by a number of performance-based tasks that teachers can use to explore student thinking in relation to key aspects of the big idea. A range of student responses is provided for each task and for each of these, there is an interpretation of what the response suggests and a description of possible teaching responses. The materials have been and are being used to support school-based coaching initiatives and state-wide professional learning programs.

The big ideas in number can be viewed in terms of the four constituent elements of horizon knowledge identified by Ball and Bass (2009) above. That is, they represent knowledge that enables teachers to locate their present instruction in the mathematical environment, they represent important disciplinary ideas and structures (e.g. place value, multiplicative thinking), they embody key mathematical practices (e.g. partitioning, generalising) and they incorporate core mathematical values and sensibilities (e.g. explanations, reasoning and problem solving).

Implications

The big ideas in number provide an organising framework for teachers to think about their task as teachers of mathematics. For some, these ideas are at the horizon, for others these ideas are emergent and/or established. Where teachers are aware of these ideas and their role in the 'mathematical landscape', they are able to 'look backwards' and plan their teaching accordingly. They are also able to anticipate students' mathematical futures, to ‘see’, ‘hear’ and act on the possibilities afforded by the insights offered by students as they grapple with the mathematics of the present.

> Knowing mathematics at the horizon gives a teacher awareness of potentialities of situations and suggests possibilities for dealing with the mathematical content being taught at a given level (Jacobsen, Thames & Ribeiro, 2013, p.9).

Far from disappearing, the knowledge derived from research and reflection on teaching practice is growing. It is this knowledge, coupled with an awareness of student’s mathematical futures that serves as horizon knowledge for teaching mathematics. However, this knowledge is not routinely shared with teachers and the mathematics horizon is only dimly portrayed in the curriculum. The big ideas in number provide a roadmap to the mathematical horizon, but it is the multi-layered conversations about student learning and inquiry-based practice[1] that are needed to undertake the journey.

Note 1. For example, take a powerful lesson such as Multo from *maths300* and collectively deconstruct the text – why did the authors decide to do this, why now, where does it lead, what learning is afforded, does it provide a roadmap to the mathematical horizon?

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