Connecting Mathematics Within and Beyond the Horizon Through Inquiry-Based Pedagogies

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Inquiry-based pedagogies have been promoted in the field for some time as a way to develop mathematical practices that students can apply in a world with uncertain and changing horizons. Inquiry is not an easy pedagogy for teachers. I am interested in how we can support teachers to develop inquiry-based pedagogies over time. Outcomes from a seven year longitudinal study with these aims will be contributed to the discussion, particularly in the connectedness of mathematics within, across, beyond its many horizons.

In this session, we are deliberating the meaning and implications of horizon knowledge. While we want to understand what it is and why it is important, we also want to go beyond this theoretical debate and think about how to make it happen. This short reflection is written as a vehicle for discussion rather than as a rigorous academic treatise. In what follows, I use mathematical inquiry to illustrate a pedagogical practice which emphasises knowledge and practice within and beyond the horizon. A second example from statistics is provided in the Appendix to challenge the destination of the content on the horizon.

Knowledge on the Horizon: What, Why and How?

In discussing horizon knowledge, researchers often focus on content beyond the curriculum: "Teachers' horizon knowledge is, for us, deeply connected to their knowledge of advanced (university or college level) mathematics" (Zazkis & Mamoto, 2011, p. 9) consisting of "advanced mathematical knowledge in terms of concepts (inner horizon), connections between concepts (outer horizon), and major disciplinary ideas and structures (outer horizon) applied to ideas in the elementary school or secondary school curriculum" (p. 13). This implies that mathematics beyond the horizon is both content-focused and static. However, this perspective of horizon knowledge is untenable for a future where the horizon is hazy. For example, the transferability of existing knowledge to new (or even familiar) contexts is problematic (Bransford & Schwartz, 1999); and there has been insufficient recognition of the role of affect and social skills in learning and *doing* mathematics in unfamiliar contexts.

Mathematical Inquiry

Mathematical practices, life-long learning (e.g., learning how to learn), social resources (e.g., collaboration, argumentation) and positive learning mindsets (e.g., efficacy, persistence) are held in regard to prepare students for the future. Inquiry-based learning has been suggested as productive in this way (Hmelo-Silver et al, 2007; O'Brien et al, 2014). Ball and Bass (2009) argue that teachers' horizon knowledge is critical for hearing the mathematical significance in students' everyday ideas, noticing opportunities for learning and making connections: all common occurrences in inquiry-based learning. This goes beyond advanced content knowledge. Mathematics is not content-free; however there is a need to bring into balance learners' experiences with mathematical structures, reasoning, and practices through addressing complex, open-ended problems.

One definition of mathematical inquiry is a process of solving ill-structured problems (Makar, 2012). In an ill-structured problem, the problem statement and/or solution pathway contain ambiguities that require negotiation (Reitman, 1965). Most problems in everyday life are ill-structured. Consider the ill-structured question, *What is the best way to travel to Brisbane CBD?* What counts as "best" is ambiguous and needs further definition as its criteria depend on the persons, purposes, context and available resources. Deciding on a method to determine the "best way" is contingent on experience and constraints.

Connectedness

Debra Panizzon's introduction to this theme on the Forum website highlights the importance of making connections for thinking about horizon knowledge. Do teachers make connections when they teach mathematics? Research (e.g., Hollingsworth et al., 2003) and findings from a recently completed longitudinal study suggest not. A study of teachers' adoption of teaching mathematical inquiry involved three phases over seven years. Teachers' pedagogies were scored using the Productive Pedagogies Classroom Observation Scheme (QSRLS, 2001). Productive Pedagogies includes four key categories of practice—intellectual quality, supportive classroom environment, connectedness, and recognition of difference—further deconstructed into 20 measurable practices (Table 1). Details and preliminary findings from the first two phases are published in Makar (2011) and a more extended publication in development.

Table 1. Productive Pedagogies

Intellectual Quality	*Supportive Classroom Environment*
Knowledge presented as problematic	Students' direction of activities
Higher order thinking	Social support for student achievement
Depth of knowledge	Academic engagement
Depth of understanding	Explicit quality performance criteria
Substantive conversation	Student self-regulation
Meta-language	Narrative
Connectedness	*Recognition of Difference*
School subject knowledge is integrated	Knowledge explicitly values all cultures
Link to background knowledge	Representation of non-dominant groups
Connectedness to world beyond classroom	Group identities in a learning community
Problem-based curriculum	Active citizenship

The four pedagogical practices under *Connectedness* emphasise making knowledge relevant, relational and transferable. These practices highlight aspects of connectedness and their implications in terms of cognition, agency and affect:

- *Knowledge integration* supports an understanding that in most problems, knowledge is not isolated (as might be experienced in textbooks). Connections can be made within and between topics in mathematics (e.g., using an area model to visualise the multiplication of binomials) or beyond mathematics between subject areas. In finding the best way to travel to the CBD, for example, there is a need to connect mathematics (e.g., duration, distance, mapping conventions, costs), English (e.g., semantics and syntax of text, multimodal representations, genre, relevance of audience) and social studies/geography (e.g., transport systems, characteristics of urban locations). When finding a best way to travel, one does not think about three separate subject areas, but applies knowledge as an integrated whole.

- *Background knowledge* invites the learner to make sense of a problem and solution using their personal and academic experiences and skills. This aspect of connectedness allows one to engage with both formal and informal knowledge. Table 2 shows coding for *Background Knowledge* on a scale of 1 (low) to 5 (high).
- *Connectedness to the world* emphasises the utility of knowledge to the wider social and political world. It evokes the relevance of knowledge for life, society and the future and engages one to imagine how the world can be influenced by knowledge.
- *Problem-based curriculum* puts problems at the heart of curriculum, including those which extend beyond a lesson, with no single correct answer, and where solutions depend on the negotiation, reconstruction and transfer of knowledge.

Table 2. Productive Pedagogy Classroom Observation Scheme:
Coding for Background Knowledge

1	No reference is made to background knowledge or experience (community, cultural or school)
2	Background knowledge mentioned, but trivial/unconnected to lesson
3	Initial reference to background knowledge; some connection to knowledge beyond school
4	Periodic reference to background knowledge; some connection to knowledge beyond school
5	Background knowledge consistently incorporated into lesson; some connection to knowledge beyond school

Teaching Connectedness in Mathematics

Connectedness is essential for developing robust relational reasoning and conceptual flexibility; if connections are not made when learning mathematics, it can create impoverished knowledge in which students demonstrate a lack of sense-making, appear unaware of mathematical structures, exhibit ineffective reasoning and ignore the reasonableness of solutions (Richland et al, 2012). Being *aware* of teaching mathematics with low levels of connectedness is not enough. What can be *done*?

One finding emerging from the analysis of our longitudinal study is that connectedness is developed in teaching mathematics through inquiry. In an analysis of lessons in the study from teachers with data collected over three years (n = 17), there were clear differences in the connectedness in regular mathematics lessons compared to mathematical inquiry lessons. Figure 1 is a stacked box plot of teachers' average scores on the four practices of *Connectedness* in a traditional mathematics lesson, their initial teaching of mathematical inquiry, and in their first, second and third year. The data suggest that *Connectedness* in a regular mathematics lesson (R) was very low ($\bar{x}$ = 1.65, s = .36), significantly improved in the first inquiry ($\bar{x} = 3.15$, $s = .68$; $t_{32} = 8.08$, $p < 0.0001$) and then continued to significantly increase as teachers gained experience from the first inquiry (A) to the third year of teaching inquiry (D) ($t_{32} = 2.33$, $p = 0.026$).

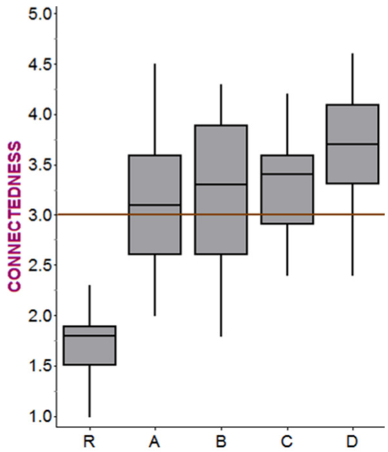

Figure 1. Distribution of teachers' average Connectedness (scale 1-5) across a regular mathematics lesson (R), first inquiry (A), and their first (B), second (C) and third (D) year of teaching mathematical inquiry.

This example assumes that horizon knowledge (disappearing as purely content and redefined to emphasise practice, lifelong learning, social resources and positive learning mindset) is improved by engaging with mathematical inquiry. It is provided for discussion as evidence of how connectedness can be improved in teaching mathematics.

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Appendix

An Example of Disappearing Horizon Knowledge in Statistics Education

In statistics education, the traditional destination towards which school and university mathematics and statistics has aimed has been inferential statistics (e.g., hypothesis testing). This focus makes sense because inferential statistics is where the power of statistics lies; it allows you to make claims about a situation (population or process) with only partial information about it (a sample). It seems obvious that one cannot do inferential statistics without a solid grasp of descriptive statistics (e.g., calculations of average and variability, representations of data) and so this has been the focus of school statistics. However, two recent developments in the field have called both the destination of statistical knowledge and the pathway towards it into question.

One major change in the field has been the recognition of *big data* (Gould & Çetinkaya-Rundel, 2014). This refers to, among other things, the ubiquitous availability and need to process streamed data (such as real-time reporting of traffic back to GPS systems or trending of tweets) or enormous databases (e.g., crimes committed in Los Angeles; consumer and market data; air traffic control; personal data collected from online interactions). Big data does not have the structure of a "random" sample or fixed population needed for the assumptions which underpin hypothesis testing. Now and into the future, citizens and workers need to be able to make sense of data that does not come in neat sample packages or is captured from a myriad of disconnected and/or dynamic sources. Big data is with us now and into the future, yet school and university topics in descriptive and inferential statistics do not necessarily transfer to contexts in which we are immersed in big data. New approaches to working with big data have been promising and yet still largely unknown.

A second development is the reconceptualization of inference. In introductory university statistics, inference is often taught as a pre-ordained procedure that indicates whether a given sample comes from an assumed population (and related questions). Because people make predictions and estimates from infancy (essentially making a claim about a situation based on incomplete knowledge of it), statistics education now recognises the value of broadening its conception of inference to include these everyday predictions. This reconceptualization, known as *informal statistical inference*, has been argued as a way to address key challenges in learning statistics (Bakker & Derry, 2011) and evidence from the field is promising in terms of children's ability to grasp it (e.g., Makar, 2014). This broader focus on everyday prediction provides a valued relevance and connection for students between learning statistics and its power to solve problems.

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