Course Design Considerations for Enhancing Pedagogical Content Knowledge in Primary Pre-Service Mathematics Teachers

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Having good pedagogical content knowledge (PCK) is an important characteristic of effective teachers. However, PCK appears to be harder to develop in primary pre-service mathematics teachers than other forms of knowledge. This paper will explore how using some of the PCK survey questions from the 'Building the Culture of Evidence-based Practice in Teacher Preparation for Mathematics Teaching' group can be used as stimulus for discussion and reflection to cultivate a better understanding of the use of manipulatives in teaching mathematics.

"Those who can, do. Those who understand, teach" (Schulman, 1986, p.14). As Lee Schulman (1986) declared in his seminal paper, "the ultimate test of understanding rests on the ability to transform one's knowledge into teaching" (p.14). Hence, there is significant pressure on those of us who prepare future teachers for practice. How do we empower primary pre-service mathematics teachers to develop the knowledge and understanding they will need, to help children make meaning of mathematics, so that they can enjoy and apply mathematics to their world? In this paper I will present aspects of my journey in course development as I continue to support primary pre-service mathematics teachers develop their knowledge and understanding to prepare them for their future classrooms.

Background

Schulman (1987) proposed there are seven categories of knowledge teachers need: "content knowledge; general pedagogical knowledge; curriculum knowledge; pedagogical content knowledge; knowledge of learners; knowledge of educational contexts; and knowledge of educational ends, purposes and values" (p.8). Pedagogical content knowledge (PCK) can be defined as knowing how to organise and present the content in ways that make it understandable to the learner. This is more than content knowledge and general teaching pedagogy. It involves knowing: how to choose, design or adapt learning activities; how to decide which representation and explanations are most appropriate; what makes some concepts more difficult to understand than others; how to avoid common misconceptions and difficulties; how to interpret and evaluate student thinking; and how to diagnose and respond to student misconceptions and errors (Goos, 2013).

Baumert et.al. (2010) in their large German study (181 teachers, 194 classes and 4353 students) of Year 10 students showed that teachers need considerable mathematical content knowledge (MCK) to have good PCK, but teachers' PCK had a larger effect, 39% of the between class variance, on student achievement than their MCK. As part of the large international Teacher Education and Development Study in Mathematics (TEDS-M) Schmidt, Houang and Cogan (2012) surveyed 2200 primary pre-service teachers in the USA. Their study confirmed that improving the quality of primary mathematics teachers could be achieved by both recruiting more mathematically capable students and through quality mathematics and mathematics pedagogy courses in their initial teacher education programs. They showed that both mathematics and mathematics pedagogy courses made a difference to improving both pre-service teachers' MCK and PCK. These findings indicate

the importance of developing both pre-service teachers' MCK and more importantly PCK, but what is the most successful way to do this?

The Journey

The 'Building the Culture of Evidence-based Practice in Teacher Preparation for Mathematics Teaching' (CEMENT) group ran their first conference in 2012. What was of most interest was the discussions on the survey questions used to determine the beliefs, MCK and PCK of Australian primary and secondary pre-service teachers, particularly the PCK questions. These questions provided an opportunity for me to determine how well my pre-service teachers were actually developing PCK and stimulated me to engage in reflective practice with my teaching. Each semester I teach Teaching Primary Mathematics P–7 to Graduate Diploma Primary pre-service teachers. These pre-service teachers complete a one year teaching qualification having previously completed an undergraduate degree. As this is their only mathematics course there is pressure for them to acquire both MCK and mathematics PCK.

The study has continued over five semesters and I have made changes to the course each year. Data collection included my reflective journal, student responses to multiple-choice PCK questions (two such examples are shown in Figures 1 and 2) with an explanation of their answer, student feedback on my teaching and student work samples.

First semester

During the first semester I wanted to determine how well the activities and discussions in the lectures and tutorials were supporting pre-service teachers to develop PCK. I knew there was improvement in their MCK as this was tested at the beginning and end of the course. PCK was measured on three separate occasions beginning midway in the course. The PCK question was given to the pre-service teachers prior to me teaching the particular topic. I then read and used their responses to inform my teaching. The pre-service teachers were then resurveyed after they had participated in the lecture and tutorial activities and discussions (and perhaps completed the readings). By comparing their initial and final responses I was able to determine changes in their thinking.

Analysing this data gave me an awareness of pre-service teachers' thinking and the fact that most appeared to lack the depth of content knowledge needed to diagnose student difficulties and misconceptions (Marshman & Porter, 2013). Pre-service teachers in general responded using the manipulatives and representations that they understood rather than addressing the student difficulty with the manipulatives that the student was using. For example in Figure 1 most pre-service teachers responded with D, the region model for fractions as "it is easier to understand."

Second and third semesters

During the next two semesters I changed the focus of some of my teaching. Now I ensured that there were regular examples of primary students' work that were analysed both in lectures and tutorials. Using small group discussion followed by whole group discussion, pre-service teachers were asked to explain, *What do you think the student was thinking when they wrote this? What does it say about what they know?* and *How might you move the student forward in their thinking?* Sometimes single examples of questions were given, sometimes multiple children's responses to the same question, or a single child's response to multiple questions.

PCK questions (including Figures 1 and 2) were given out during the semester prior to and after teaching and the responses analysed. Pre-service teachers were appreciative of the opportunity to see and discuss samples of student work as one pre-service teacher commented on her survey in response to which activity had helped and how?

> "I found the slides in the lecture were good because it's interesting seeing students working out + how they come to that conclusion."

Another responded "the Blackboard example" as a number of other examples of student work with explanations had been placed on the Learning Management System.

Pre-service teachers found the fraction PCK question in Figure 1 the one most difficult. Their responses showed more pre-service teachers now were beginning to recognise that one needed to work using the same manipulatives as the student, and that the student was having difficulty with the set or collection model of fractions. Some examples include:

> Pre-service teacher A: "Using the same manipulative, shows $\frac{1}{4} + \frac{1}{4} = 2$ <u>out of</u> four."
>
> Pre-service teacher B: "I chose C because she is struggling with the concept that the whole doesn't change. By maintaining the whole and adding the parts in a similar visual representation (so D & E are not sufficient) as you question you can clarify her understanding. A would reinforce the whole changing in her mind even though it is correct. E reinforces her misconception."

If examples of primary students' work were helping some students develop PCK maybe if I was more explicit with my thinking I could help more pre-service teachers?

Fourth and Fifth Semesters

As I was still unhappy about the general level of pre-service teacher responses I decided to include some of the PCK questions as lecture slides. Small group discussion was followed by whole group discussion, during which I made my thinking explicit, used manipulatives to demonstrate my chosen examples and why I wouldn't use others.

For example with the question in Figure 1 we discussed that perhaps the student had used the counters to show $\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$ because that was the only model that she could use to explain her understanding, and perhaps if she had used D (the region model) then she would not get $\frac{2}{8}$ which is what her 'rule' said was the answer. I explained sometimes people could hold conflicting conceptions and that unless the misconception was addressed where it actually was there were no guarantees that there would be a change in understanding. To many pre-service teachers this was puzzling so I explained that although D, the region model was most popular and could show the student that $\frac{1}{4} + \frac{1}{4} \neq \frac{2}{8}$ unless we address the difficulty that the whole was four counters, the student may not change their thinking.

Another interesting discussion was held around the slide in Figure 2. In this case the student is in Year 2 which can be challenging for pre-service teachers who have not completed a practicum in an early years classroom or don't have young children. For some pre-service teachers it was surprising that you would ask children to measure a desk that was not a whole number of popsticks, (that the real world is messy) and if it wasn't a whole number then why not just tell them it is 'close enough'. Other pre-service teachers had not considered that D using a smaller unit to fill the missing part might lead to misconceptions. One pre-service teacher demonstrated that she needed four cubes to complete the measurement and that there were seven of the cubes we were using in a popstick so the extra was four sevenths. She took some convincing first that not many Year 2 students would understand four sevenths but also that she had changed the units she was

using to measure! Discussions also explored when you might ask a student to measure 'more carefully', but how you could do it with the student realising that there should be 'no gaps or no overlaps' when laying the popsticks end-to-end.

By explicitly making my thinking visible to pre-service teachers during discussions of student work samples and these PCK questions I think I am seeing an improvement in pre-service teachers' PCK. One pre-service teacher even referred to this second example in their course feedback. "To have …teaching pre-service teachers to count pop sticks is a most extraordinary gift … thank you. [The] subject is taught explicitly to identify misconceptions amongst students, to show them many strategies to succeed with primary mathematics concepts, to identify curriculum strands and apply knowledge to teaching scenarios.…"

Other pre-service teachers are now referring to it in more detail in their critique of a scenario. In the example below, from a, three aspects are identified: the misconception, how it could be addressed and age appropriateness.

"The following misconceptions are identified:

a) The fractional expression of ½ of 40 is incorrectly written in a divisor/dividend format. I would use a teachable moment in class for all students to benefit from correcting this misunderstanding, as the curriculum requires student knowledge of fractions in ACMNA077 (ACARA, 2014);
b) Square roots are incorrectly expressed. As square roots are not introduced in the curriculum until year 7, I would address the misconception with the individual student, but would not introduce the curriculum content at this stage. …" (pre-service teacher's critique)

Conclusion

As pre-service teacher educators we want to produce highly professional graduates. We cannot possibly teach these pre-service teachers everything they will need to know but we do want them to be able to continue their learning on their own. This paper has described my journey as I have continued to develop Teaching Primary Mathematics P-7 to support pre-service teachers develop their PCK. The focus of this reconceptualisation has been including many samples of primary students work and the PCK questions from the CEMENT project and most importantly explicitly sharing my thinking with my students. The challenge is that curriculum development is never complete so my journey continues….

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Appendix

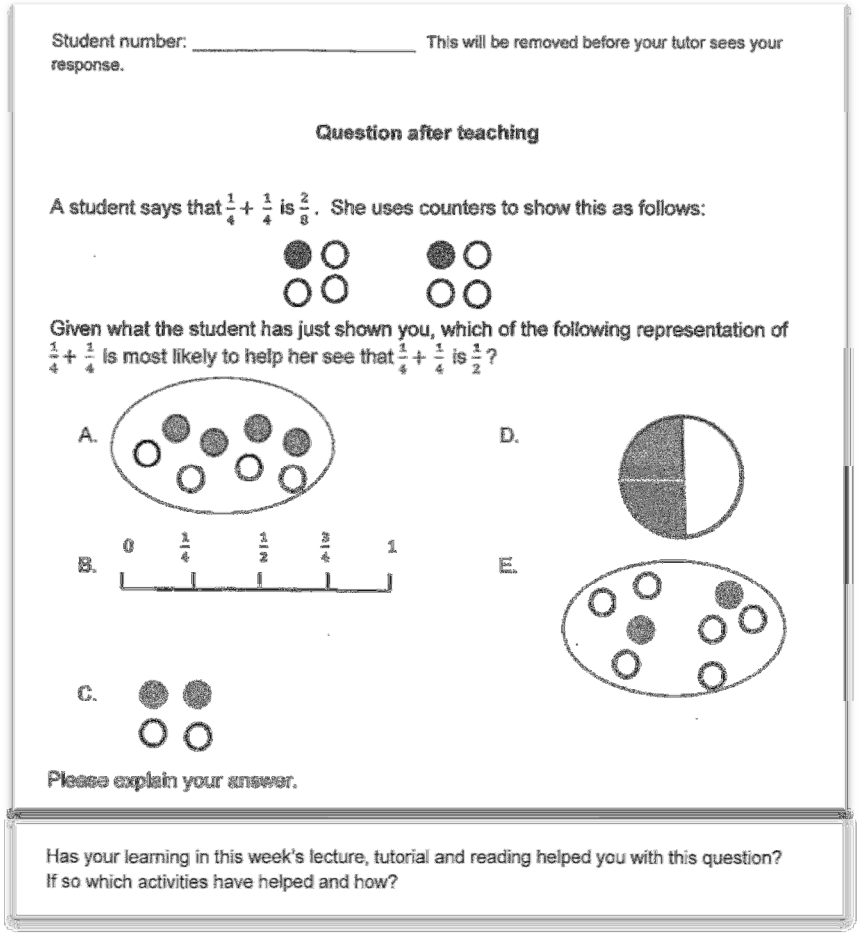

Figure 1. The fraction PCK question as it was given to pre-service teachers after teaching

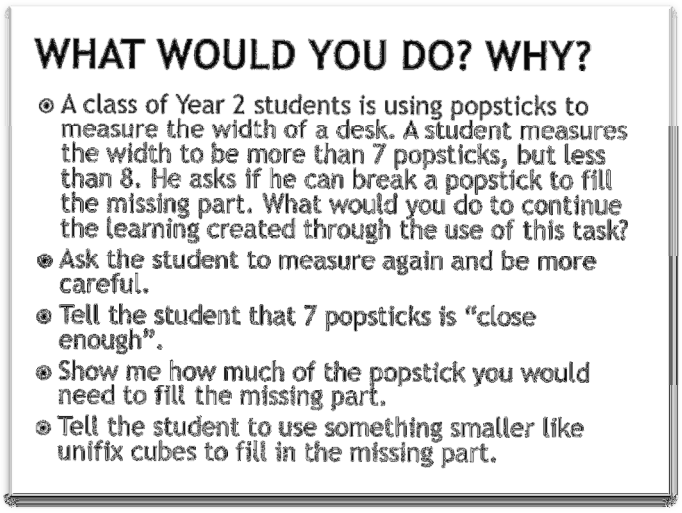

Figure 2. The measurement PCK question used as discussion in a lecture