Categorising the Mathematical Content Knowledge Pre-service Teachers Develop during Teacher Education

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The mathematical content knowledge (MCK) a teacher brings to the classroom is important as it underpins the decisions they make when teaching. Targeted opportunities to learn different categories of MCK during program experiences may assist pre-service teachers to deepen their understanding of MCK for primary mathematics teaching. This paper reports on findings from a four-year longitudinal study of the development of primary pre-service teachers MCK during program experiences. The results identified the development of *foundation knowledge* as fundamental and should be established at the beginning of their program if pre-service teachers are to extend their MCK and *horizon content knowledge*.

Horizon content knowledge is a category of subject matter knowledge and is a peripheral vision that informs mathematics teaching practice (Ball, Thames, & Phelleps, 2008). Understanding the MCK that pre-service teachers develop during their teacher training is vital for program design and developing their *horizon content knowledge*. This paper presents different categories of MCK and was informed by an historical overview of theoretical frameworks and the findings of a four-year longitudinal study (Livy, 2014). The 17 pre-service teachers in the study demonstrated different categories of MCK during their program. When observed teaching in the final year of a Bachelor of Education program the pre-service teachers were not yet demonstrating *horizon content knowledge*.

From a research perspective, frameworks of teacher knowledge can assist with deepening understanding of the different categories used to describe this knowledge, as well as its use for *effective* teaching of mathematics (Bobis, Higgins, Cavanagh, & Roche, 2012). In brief, Shulman (1987) characterised teachers' content knowledge as concerned with expertise in the particular discipline being taught. Building on the seminal work of Shulman and others (e.g., Chick, Baker, Pham, & Cheng, 2006; Ball, Thames, & Phelps, 2008; Rowland, Turner, Thwaites, & Huckstep, 2009) researchers have developed frameworks as a means for understanding the complex relationship between the types of knowledge required for mathematics teaching. The domains of Mathematical Knowledge for Teaching included three categories of subject matter knowledge: *common content knowledge*, *specialised content knowledge* and *horizon content knowledge* (Ball et al., 2008). *Horizon content knowledge* is when a teacher demonstrates understanding of the complexities of mathematical topics, has advanced knowledge, broad understanding of mathematical ideas and connections, and links their *common content knowledge* with curriculum that their students know and will know in future years (e.g. Ball & Bass, 2009; Ball et al., 2008). Ma (1999) described accomplished teachers as demonstrating profound understanding of fundamental mathematics (PUFM) demonstrating *breadth*, *depth* and *thoroughness* of their MCK. The Knowledge Quartet framework identifies four categories of MCK: *foundation knowledge* what teachers know and believe*; transformation* choice of examples and representations; *connection* between procedures and decisions about sequencing of lessons; and *contingency* responding to student questions (Rowland et al., 2009). Chick et al., (2006) combined MCK and pedagogical content knowledge in their pedagogical content knowledge category of content knowledge in a pedagogical context,

listing five categories for classify different aspects of how a teacher may demonstrate their MCK including: *methods of solution* a method for solving a problem; *procedural knowledge* where conceptual understanding need not be evident; *mathematical structure;* and *connections,* making connections between concepts and topics. Furthermore it is expected that Australian teachers know the content they teach (Australian Institute for Teaching and School Leadership (AITSL), 2011).

Method

See Livy (2014).

Mapping MCK

MCK is defined as the knowledge of mathematics pre-service teachers rely upon, develop and demonstrate during program experiences including coursework completed at university and practicum teaching in primary schools.

The following is a suggested sequence for developing the different categories of MCK:

1. Methods of solution
2. Procedural knowledge and common content knowledge,
3. Foundation knowledge
4. Breadth and depth; transformation; connection; contingency; *specialised content knowledge*
5. Effective numeracy teacher
6. PUFM

A teachers' common content knowledge enables them to know the mathematics they teach (Ball et al., 2008). When first solving problems pre-service teachers may know how to demonstrate one or more *methods of solution* but may not know how to use material to model the solution or know different strategies students use when responding to similar mathematical problems. At the beginning of their program pre-service teachers often relied on *procedural methods* having difficulties with *common content knowledge* and needed to consolidate their *foundation knowledge* (Livy, 2014). They relied on procedures rather than demonstrating understanding and lacked mathematical language therefore were not demonstrating categories of *foundation knowledge*.

The first three categories *methods of solution*, *procedural knowledge* (Chick et al., 2006) and *common content knowledge*, (Ball et al., 2008) describe MCK pre-service teachers should have learnt during their own schooling. However responses to a Mathematical Competency Skills and Knowledge Test in second-year identified that many pre-service teachers needed to revise their *common content knowledge* required for test items ranging in difficulty from Year 5 to Year 8 mathematics (ACARA, 2013). For example the most difficult number items that may have relied on *procedural knowledge* included using proportional reasoning for calculating scale, distance and capacity items, and conceptual understanding when selecting composite numbers from a list of two-digit numbers as well as generating an appropriate model for division of common fractions.

The findings of the longitudinal study (Livy, 2014) identified that *Foundation knowledge* was important and should be developed during first and second years, so that pre-service teachers can extend their MCK in later years of the other categories of MCK special to teachers. *Foundation knowledge* concerns pre-service teachers readiness for their teaching role (Rowland, Huckstep, & Thwaites, 2005) and assists teachers to make

decisions when teaching, includes beliefs about mathematics and underpins the other three dimensions of the Knowledge Quartet framework (Rowland et al., 2009). In this sequence *Foundation knowledge* includes *methods of solution*, and *procedural knowledge* and *common content knowledge*

The next stage of development included evidence of *breadth* and *depth* (Ma, 1999), *transformation*, *connection* and *contingency* (Rowland et al., 2009) and *specialised content knowledge* (Ball et al., 2008). These categories were considered interrelated when coding pre-service teachers' MCK, especially during pre-service teachers' practicum experiences when they were observed teaching a primary mathematics.

Depth and *breadth* (Ma, 1999) of MCK were identified by comparing the distribution and year levels that pre-service teachers experienced in schools during their practicum placements. When pre-service teachers planned and taught a series of lessons they had the opportunity to demonstrate their *breadth* of mathematical knowledge building on from one lesson to the next or from one topic to the next, which was also making *connections*. *Depth* of MCK could be developed when pre-service teachers practised their teaching across different year levels, by demonstrating how well they could *transform* what they knew in ways that made knowledge accessible to primary students. For example, using base ten blocks when modelling addition of two-digit numbers.

The program structure assisted pre-service teachers to develop *breadth* of experience for each year of the program when in schools. They experienced a total of 102 days in primary schools during first (n=20), second (n=32) and fourth years (n=50), assisting or teaching with their mentor teacher (classroom teacher) on Tuesdays as well as one week block placements usually with a different year level for each year of their primary school placements. Third-year included experiences in secondary schools (n=42) when pre-service teachers' developed teaching skills for their secondary discipline. Although pre-service teachers experienced different year levels during the three placements in primary schools some pre-service teachers did not experience lower, middle and upper year levels therefore limiting their *depth* of experience and development of MCK.

Specialised content knowledge is a unique kind of content knowledge special to teachers (Ball et al., 2008). Pre-service teachers beginning to demonstrate *specialised content knowledge* could demonstrate developing *breadth* and *depth* of MCK; *transformation*, making *connections* and dealt with *contingencies*. When pre-service teachers planned and taught primary mathematics lessons they demonstrated their *specialised content knowledge* when they recognised the links within a topic, understood the difficulties of a topic and order for teaching different concepts. They were able to *transform* their MCK by using appropriate teaching strategies or representations and *connections* as well as *breadth* of MCK. In this study there was little evidence of *contingency* but some pre-service teachers were able to respond to unexpected student questions during a lesson.

The highest level of MCK among the longitudinal pre-service teachers was *specialised content knowledge*. When observing the longitudinal study participants teaching a primary mathematics lesson during their fourth year, four demonstrated *specialised content knowledge*. These pre-service teachers provided evidence of their *breadth* and *depth* of MCK and readiness to teach primary mathematics. However, only three pre-service teachers were coded SCK after comparing their fourth-year lessons and responses to a MCK interview task because of a reliance on *procedural knowledge*.

In fourth-year Kerri (pseudonym) was observed teaching Year 5 & 6 students. The students were discussing different responses to a numeracy test. Kerri could rely on her

foundation knowledge when teaching and demonstrated her *breadth* of MCK by making *connections* of different topics discussed during the lesson. She also assisted students to develop their understanding, demonstrating her *specialised content knowledge*. During an explanation of a geometry problem that included counting the number of rectangles needed to make a hexagonal prism Kerri relied on her SCK including choice of teaching strategies. Kerri drew a net of prism to assist the students to check the correct response of six rectangles and visualise the problem.

An *effective* teacher demonstrates more than *common content knowledge*, and relies on important categories of subject matter knowledge (Ball et al., 2008) or *foundation knowledge*, as well as beliefs about mathematics and pedagogical content knowledge (Rowland, et al., 2009). A characteristic of an *effective* mathematics teacher is *breadth* and *depth* of knowing school mathematics (Ma, 1999). *Breadth, depth, transformation, connection* and *contingency* will also assist pre-service teachers to develop their *horizon content knowledge*. None of the pre-service teachers in the study demonstrated PUFM or *horizon content knowledge* and these categories might not be developed until teachers begin teaching or have extensive primary mathematics teaching experiences.

The pre-service teachers in this study were not yet demonstrating *horizon content knowledge* as they did not always know how to extend their MCK during a teaching situation. The program structure did not provide *breadth* and *depth* of experiences for developing *horizon content knowledge.* A factor contributing to these results may be that during practicum experiences pre-service teachers missed teaching experiences across different year levels including upper primary (Livy, 2014). Future practicum experiences could also consider teaching experiences in lower secondary mathematics classes.

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